

## LOCAL HEAT TRANSFER RATES FROM TWO ADJACENT SPHERES IN TURBULENT FLOW

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**Abstract**—The interactions between a pair of spheres in a crossflow orientation was studied in a vertical wind tunnel. The flow Reynolds number was approximately  $10^4$  with a turbulence intensity of 1.1%. The spacing between the sphere centres was varied from 1.04 to 2 diameters. Both the local surface pressures and heat transfer coefficients were measured. The measurement technique allowed the fluctuating component of the heat transfer to be measured as well as the time averaged values. The present results indicate the formation of a jet-like flow between the spheres. At close spacing, this jet remained attached to one of the spheres giving rise to crossflow forces. At very close spacings, the increased shear stress in the region between the spheres resulted in a decrease in the strength of the jet and the magnitude of the crossflow forces.

### 1. INTRODUCTION

THE FLOW around suspensions of particles has been of interest since the turn of the century but, as yet, it is not well understood. The usual analysis of such systems is based on flow models developed for a single sphere. At low Reynolds numbers, the Stokes 'creeping flow' equation provided a satisfactory description. By including inertial terms in an asymptotic expansion of the solution, the formation of a vortex ring wake can be predicted. This model, however, breaks down when the Reynolds number increases beyond a value of 40. At higher Reynolds numbers, over the leading part of the sphere, boundary layer models may be used successfully, but these break down when the layer leaves the surface of the sphere. The flow in this wake regime has been the subject of many experimental studies.

The flow around the simplest form of cluster, a single pair of spheres, has been the subject of many studies based on the classical simplifications of the Navier-Stokes equations. The first solution for the potential flow around two similar spheres were presented by Hicks [1] in 1880. The solution was built up by superimposing a series of simple flow singularities. Basset [2] and Herman [3] expanded the integral of the surface harmonics to obtain the kinetic energy of the fluid. The velocity potential in the neighbourhood of the spheres was then expanded by the 'Addition Theorem'. In calculating the interaction between particles in dilute suspensions, Peskin [4] used an equation derived by Konig [5]. This model, however, included only first-order terms. The spheres appear to attract each other regardless of the orientation with respect to the flow since the forces are directed along the displacement vector between the spheres. Latta and Hess [6] used the 'Inversion Theorem' to solve the case of flow past two spheres in perpendicular flow. The sphere boundary was transformed into an infinite plane one diameter away from the plane of symmetry. The velocity potential was then expanded as harmonic functions along the plane. To summarize these

potential flow solutions it is generally predicted that an attraction force between the spheres exists. Because of the symmetry of these solutions, no form drag was predicted in potential flow.

The first analysis of the creeping flow around two spheres was carried out by Smoluchowski [7]. The drag forces against the direction of motion were shown to be smaller than those predicted by Stokes law. Analyses of this type have also been carried out by Burgers [8, 9] and by Faxen and Dahl [10].

The exact solutions for two equal spheres whose line of centres was perpendicular to the flow were developed by Goldman and Brenner [11] and by Wakiya [12] using bispherical co-ordinates. The drag in this configuration decreased as the spheres approached. However, additional forces causing the spheres to rotate were predicted. When boundary conditions which permitted the rotation were used, the angular velocity of the spheres was obtained.

These analyses, based on the creeping flow equations, did not allow the consideration of any inertial effects. Oseen [13] extended the analysis of Smoluchowski [7] to higher Reynolds numbers through an asymptotic expansion technique. The effect of inertial force on two equal spheres was a tendency to separate them.

Several measurements of the terminal velocity of pairs of equal spheres falling in viscous fluids were carried out. Perhaps the first sufficiently accurate data was published by Evesen *et al.* [14]. The results were compared with the analyses of Smoluchowski [7], and Burgers [8]. The prediction diverged upwards at a separation of 1.8 diameters and was followed by that of Burgers at 1.3 diameters. This was attributed to the series function error and in some part to the presence of inertial effects.

Matthews and Smith [15] observed that spheres in a non-axisymmetric configuration rotated and were subject to inertia effects. These inertia effects, attracting and repelling forces, were similar to those predicted by Oseen [13]. Jayaweera *et al.* [16] carried out a series of sedimentation experiments involving clusters of

between two and seven spheres falling in a viscous fluid. When two equal spheres were dropped side by side, no separation or rotation was observed for Reynolds numbers below 0.03, however an inwards rotation was observed for Reynolds numbers greater than 0.05. The rate of this rotation increased with the Reynolds number, but decreased with increasing separation distance.

The drag forces acting on a pair of spheres at high Reynolds numbers were measured by Tsuji *et al.* [17] and Lee [18]. As the spacing between the spheres was decreased, the drag coefficient rose to approximately 1.5 times that of a single sphere. In addition, Lee reported a separation force which also increased as the spacing decreased.

In summary, these studies reveal much of the flow behaviour at relatively low Reynolds numbers, where viscous forces dominate. At high Reynolds numbers which are often encountered in practice, inertial forces dominate the free-stream while the viscous forces are confined to the boundary layer. It has been established that the boundary layer and wake behaviour of a single sphere observed in turbulent flow are greatly influenced by the flow conditions. Therefore it may be expected to have a great influence on the flow interaction between two spheres and consequently the local heat and mass transfer rates. No adequate mathematical models of these phenomena, particularly the wake behaviour, have been developed. Hence, the flow phenomena as well as heat transfer rates must be studied experimentally. The present study represents an experimental investigation of two adjacent spheres in turbulent flow.

## 2. EXPERIMENTAL DESCRIPTION

The wind tunnel used in this study was a low speed, closed loop system. The features of this system are a flat velocity profile in the test section, a low level of background turbulence and good temperature control. The flow conditions were measured by hot-wire anemometry.

The spheres were supported by crossflow stems whose diameters were 0.18 that of the spheres. The mountings allowed the spheres to be accurately positioned in the test section and rotated about the stem axis.

The sphere models used for the local surface pressure measurements were two identical brass ball-bearings, 2.705 cm in diameter. The surface pressure was sensed through a hole 0.709 mm in diameter, located at  $90^\circ$  to the stem axis. The pressure was measured to an accuracy of 0.01 mm water by a projection-type micromanometer. The surface pressure measurements were taken at  $10^\circ$  intervals on the sphere surfaces and integrated to give the overall drag coefficients.

The heat transfer measurements were obtained by a technique analogous to hot-wire anemometry. Two small platinum films were deposited onto each of two hand-blown fused-silica spheres. The spheres were of equal diameter (2.654 cm with a measured standard

deviation of 0.6%). The individual films were heated to the same temperature electrically by a constant temperature anemometer circuit. The inner film served as the local heat transfer sensor while the outer film minimized the tangential heat loss through the silica surface. The remainder of the sphere surfaces were heated to approximately the same temperature as the platinum films by internal heaters consisting of 10 m of number 34 gauge magnet wire. The void space around the wire was filled with a 55% slurry of powdered aluminum in ethylene glycol.

A complete description of the experimental equipment and the derivation of the necessary relationship have been presented elsewhere [19, 23].

## 3. DISCUSSION OF EXPERIMENTAL RESULTS

The present experimental study of the flow behaviour around two spheres in the perpendicular configuration consists of measurements of the local surface pressures and both the average and fluctuating components of the local heat transfer. The surface pressures were measured at Reynolds numbers of 10 800 and 7950. The heat transfer rates were obtained at a Reynolds number of 8800. The free-stream turbulence intensity was approximately 1.05%. Since the flow over the surface of each sphere was not symmetrical about an axis through the centre of each sphere, the local measurements were taken in a series of traverses along several meridians on the sphere surfaces. The overall drag coefficients were integrated from the local surface pressure data at the higher Reynolds number shown in Fig. 1. The spacing

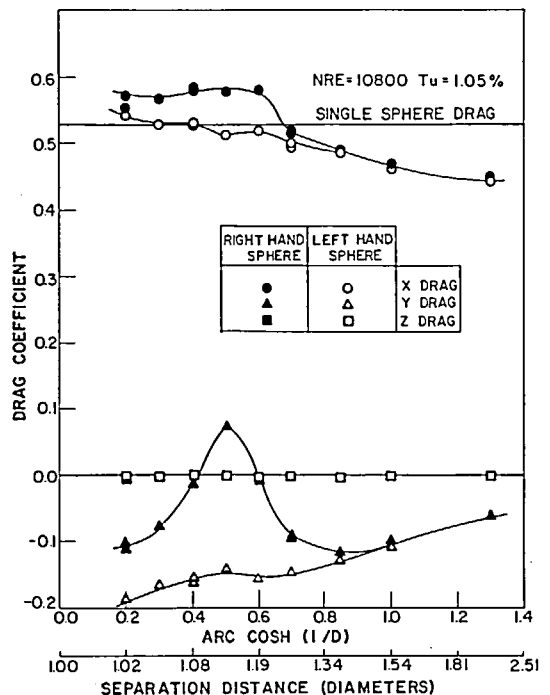


FIG. 1. Overall drag coefficients of two spheres in perpendicular configuration.

parameter used was that of the bispherical co-ordinate system. The scale expansion inherent in this system allowed the interactions between the two spheres at close spacing to be shown more clearly. The integrations were performed such that the forces on the spheres were resolved into three components, each expressed in terms of a drag coefficient. The component directed in the mean flow direction was the drag analogous to that of a single sphere. The second, directed inwards along the line of centres, represented the attraction between the two spheres. The third component, perpendicular to the other two, was found to be negligible.

When the spheres were far apart, the drag and repulsion forces shown in Fig. 1 acted equally on both spheres. As the spheres were moved closer than 1.34 diameters between centres, the flow appeared to adhere to one of the spheres. This resulted in unequal forces acting on the two spheres. The measured forces were, therefore, resolved into four components representing symmetrical and antisymmetrical effects. These components, expressed in terms of drag coefficients, are shown in Fig. 2.

The resolved forces in the flow direction consist of the drag which acts equally on each sphere and the shear which acts oppositely on each sphere. The equal of symmetric forces were calculated by averaging the individual measured drag coefficients and the opposite or antisymmetric forces by taking half of the difference between the individual values. The repulsion and side-slip forces are analogous to the drag and shear forces but were directed inwards along the line of centres between the spheres. The choice of these components was arbitrary. However they represent the simplest form which illustrates the flow behaviour.

The overall drag shown in Fig. 2 was approximately

16% lower than the corresponding drag on a single sphere. The drag increased as the spheres were brought together, becoming essentially constant at a value slightly higher than the single sphere drag. The repulsion increased as the spheres approached, from 13.5% of the drag at a spacing of 1.97 diameters to 24.5% of the drag at 1.34 diameters. The antisymmetric forces were negligible in this range of spacings.

When the spheres were brought closer together, the flow appeared to adhere to one sphere, deflecting the flow and giving rise to the antisymmetric forces. These forces rose to a maximum at a spacing of 1.13 diameters. The side slip was the dominant feature of the interaction, reaching a maximum of 19% of the drag. The repulsion between the spheres decreased to a minimum at this point. At yet closer spacings, the repulsion force increased quickly while the antisymmetric forces decreased. Apparently the flow resumed its symmetrical character at very close spacings.

The apparent attachment of the flow between the spheres to one of the spheres did not favour a particular sphere but was observed to alternate between the spheres. The change of the attachment from one sphere to the other was not frequent, but occurred randomly at time intervals ranging from about 1 min to 1 h. When the change occurred, the local surface pressures were observed to be the same as those before the change but on the opposite spheres.

If the flow in the test section was disturbed by the insertion of an obstacle upstream from the spheres, the attachment of the jet could be moved from one sphere to the other. This suggests that turbulent eddies from the free-stream are responsible for the observed shifts in the flow attachment.

The overall Nusselt numbers are shown in Fig. 3. The value on the left-hand sphere was approximately 20% higher than the overall Nusselt number of a single sphere. A dip was noted in the range of spacings as the

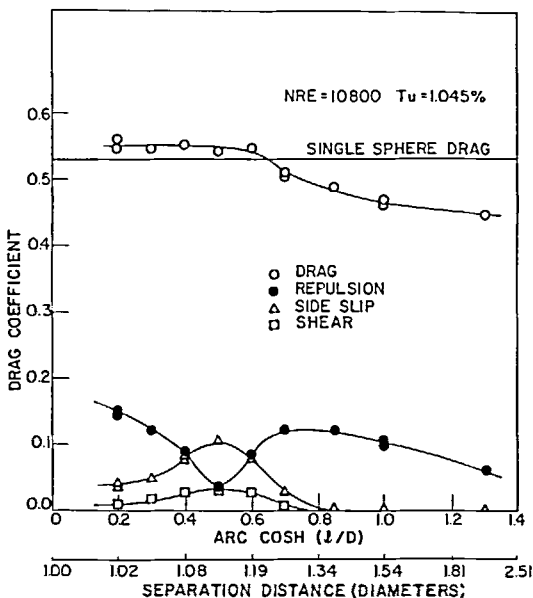


FIG. 2. Resolved drag coefficients of two spheres in perpendicular configuration.

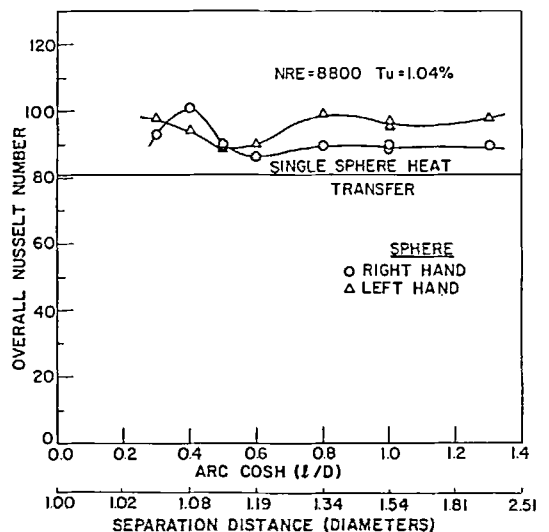


FIG. 3. Overall Nusselt numbers from two spheres in radial orientation.

maximum antisymmetric forces were observed. The heat transfer coefficients from the right-hand sphere were lower than those of the other sphere, approximately 10% higher than the single sphere value. The heat transfer from the right-hand sphere rose at the same spacing as the minimum heat transfer from the other sphere and reached a maximum at a spacing of 1.08 diameters. This increase appeared to correspond to the reduction of the flow deflection.

The flow interactions observed in this study were similar to those predicted for creeping flow. The drag on the spheres was less than that on a single sphere and a repulsion force between the two spheres was observed. The creeping flow models, however, predict a reduction in the drag as the spheres are moved closer together. The opposite was observed in the present study. The antisymmetrical forces resulting from the attachment of the flow between the spheres to one of the spheres was not predicted by the creeping flow models, nor was it observed experimentally at low Reynolds numbers. The mechanism of these interactions is related to the separation of the boundary layers from the spheres.

The surface pressure distributions on the two spheres are shown in Fig. 4. When the spheres were far apart, at

a spacing of 1.97 diameters, the surface pressures on both spheres were approximately the same. The pressures on the inward side of the spheres (i.e. the surface facing the other sphere) were higher than those on the opposite side (facing away from the other sphere). This pressure difference resulted in the net repulsion between the two spheres. As the spheres were brought together, the pressure on the inward face increased as the surface friction and the blockage due to the wakes caused a greater deceleration of the flow between the two spheres. The pressures on the outward surfaces were not greatly affected.

When the spheres were brought closer than 1.13 diameters, the increase of the pressure on the inward surfaces of both spheres continued. The flow between the spheres, however, adhered to the right-hand sphere resulting in reduced surface pressures on the rear part of the inward surface of that sphere. The pressures on the corresponding rear part of the surface of the other sphere were somewhat higher than those observed before the attachment. The magnitude of these pressure changes increased as the spheres were brought closer, up to a spacing of 1.05 diameters.

The unbalanced pressure forces caused by the flow

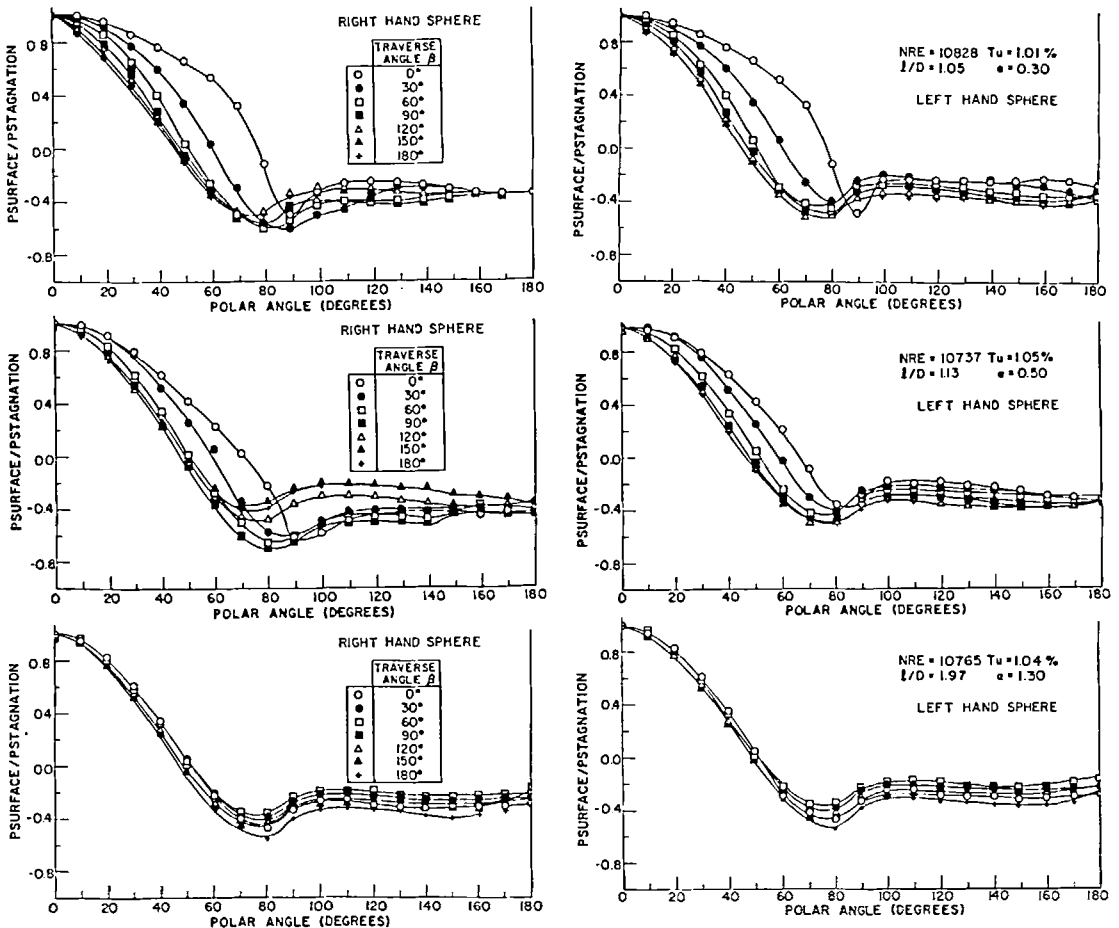


FIG. 4 Local surface pressure on two spheres in perpendicular configuration.

attachment increased as the spheres were moved from a spacing of 1.93 diameters to one of 1.05 diameters. These forces became larger than the repulsion forces resulting from the increase in pressure due to the deceleration of the flow between the two spheres. As shown in Fig. 1, the reduced pressure due to the flow separation became greater than that causing the repulsion, changing the direction of the overall lateral force on the right-hand sphere.

At closer spacings, the attachment effect became weaker as the pressures observed in the attachment region of the right-hand sphere were not as low as those observed at a spacing of 1.13 diameters. The increase in the pressure between the spheres caused by the flow deceleration was much more pronounced. Again the pressures on the rear part of the left-hand sphere were not greatly changed as the spheres were brought together.

The mechanism of the attachment of the flow between the spheres to one of the spheres was analogous to the Coanda effect [20, 21] where a jet follows a curved wall. The flow constriction between the two spheres produced a jet-like flow when the boundary layers separated from both spheres. This jet was, however, a 3-dim. structure which widened as the

distance between the sphere surfaces increased, that is, as the angle from the plane parallel to the mean flow and the line of centres increased. The flow in the jet was faster than that in the wake, hence the pressure in the jet was lower and entrainment of the wake fluid into the jet occurred. When the jet was pushed toward one of the spheres by the turbulence, the wall inhibited the entrainment of fluid into the jet, resulting in a reduced pressure on that side of the jet. The resulting unbalanced pressure forced the jet to the surface of the sphere until the flow detached due to the adverse surface pressure gradient and the wake flow.

The initial displacement of the jet towards one of the spheres probably occurred as the result of the free-stream turbulence. Once attached, the jet configuration was quite stable, however a large eddy from the free-stream turbulence could interact with the jet sufficiently to cause it to leave the sphere and become attached to the other sphere. The nature of the interaction between the free-stream turbulence and the jet flow was not studied in the present investigation.

The flow patterns were observed to be equivalent in the two attachment configurations. The surface pressures on the sphere with the attached jet did not depend on which physical sphere was involved. This

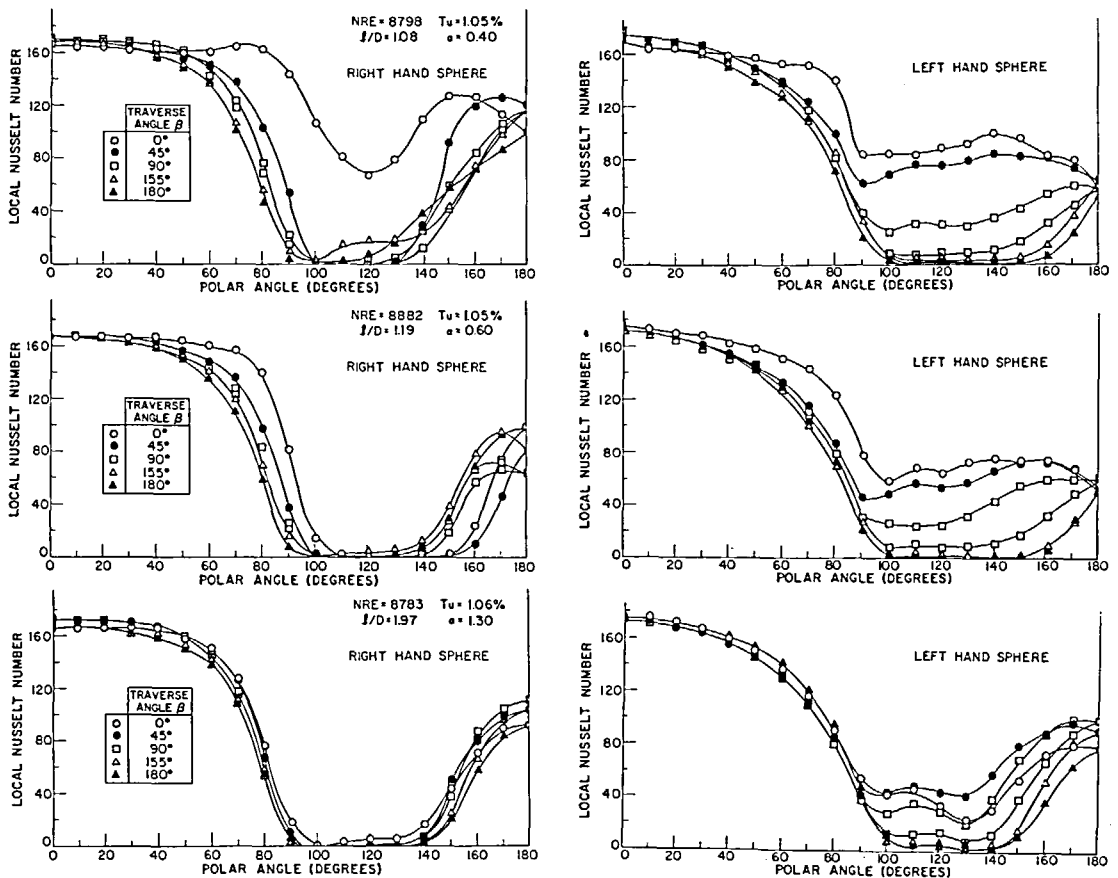


FIG. 5. Local Nusselt numbers from two spheres in perpendicular configuration.

was also observed for the sphere without the attached jet. This behaviour was analogous to that of a bistable wall attachment logic device used in fluidic control systems [22].

The interaction between the two spheres was also shown by the local heat transfer data presented in Fig. 5. At the furthest spacing, 1.97 diameters, the flows on the two spheres were not completely similar. The heat transfer from the laminar boundary layer region was slightly higher than that observed on a single sphere. The local Nusselt numbers on the inward surfaces of the spheres were higher than those on the surfaces facing away from the opposite sphere. This increased as the spheres were moved closer due to the greater deceleration of the flow between the spheres. The altered pressure distribution resulted in a thinner thermal boundary layer.

The heat transfer from the turbulent boundary layer and wake regions showed the most striking difference. The heat transfer from the right-hand sphere was similar to that from a single sphere although the coefficients from the wake were slightly higher than those of a single sphere. The values from the left-hand sphere varied greatly with the traverse angle and were higher than those from a single sphere. Although part of this increase may be attributed to the temperature history effect observed in the single sphere trials, the effect of the jet attachment was clearly noted at these spacings. As the spheres were moved closer, the

turbulent separation point on the inward surface moved forward. At 1.19 diameters spacing, the turbulent boundary layer on the left-hand sphere was observed only on the outward facing surface of the sphere. It would appear that the deflection of the flow caused by the attachment of the jet to the right-hand sphere caused the turbulent separation from the inward surface to occur earlier, and at close spacings, to prevent the reattachment of a turbulent boundary layer after the separation of the laminar boundary layer. This behaviour continued as the spheres were moved to the smaller spacing.

The heat transfer from the right-hand sphere showed considerable change when the jet attachment was strong. The heat transfer from the laminar boundary layer increased as the spheres were moved together, particularly on the inward side of the sphere. The separation of the laminar boundary layer was delayed by the attachment of the jet. When the spheres were moved to a spacing of 1.08 diameters, the turbulent layer was not observed on the innermost part of the surface. The laminar layer separated directly into the wake at an angle of  $120^\circ$  from the front stagnation point. At the closest spacing, 1.04 diameters, the turbulent boundary layer was again observed. The heat transfer from the laminar boundary layer decreased sharply at the separation point.

The flow on the outward surface was similar to that on the inward surface of the opposite sphere. The

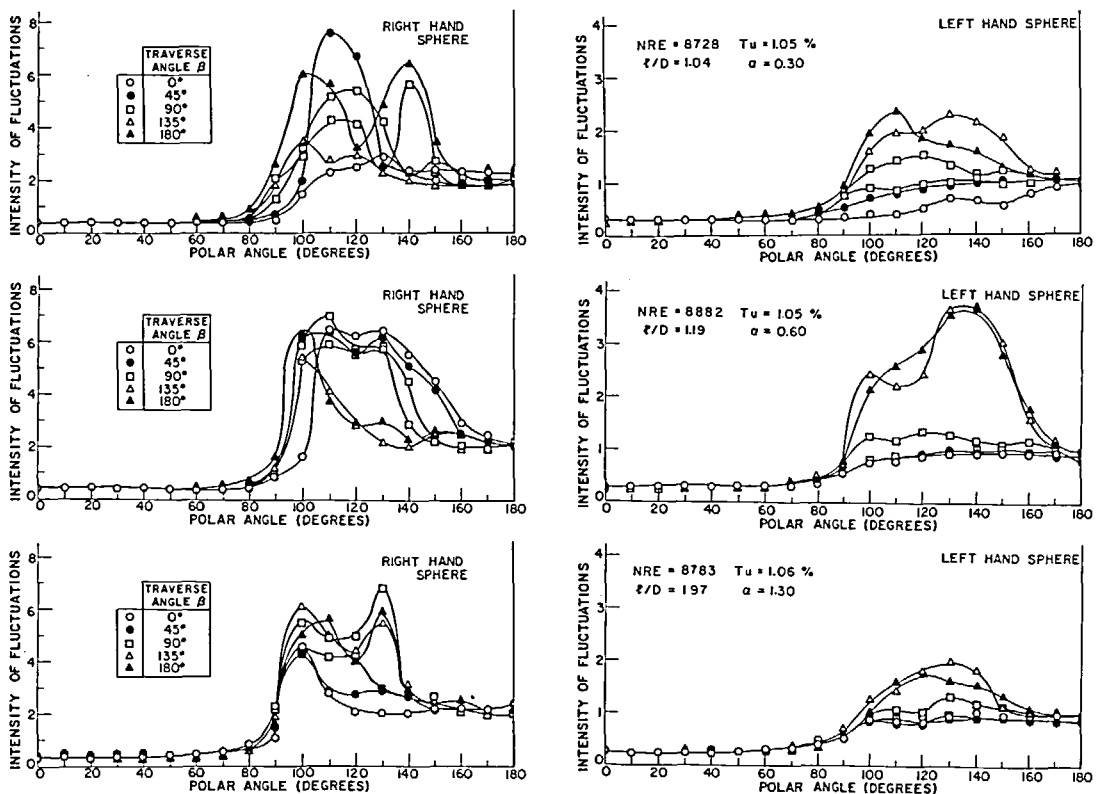


FIG. 6. Local heat transfer fluctuations on two spheres in perpendicular configuration.

turbulent boundary layer separated much earlier from the sphere surface. This was caused by the shift in the wake resulting from the lateral flow developed by the attached jet.

At the closest spacing, the flow on the right-hand sphere was similar to that at much wider spacings, although the heat transfer from the wake was large. The left-hand sphere, however, did not return to the flow configuration at wider spacings, but was similar to that with the strong jet flow. It would appear that although the jet attachment effects were not as pronounced on the right-hand sphere, the effects in the space between the spheres remained significant. The decrease in the strength of the jet was due to the increased blockage of the flow between the spheres by the combined wakes.

A qualitative description of the boundary layer behaviour may be developed through an examination of the intensity of the fluctuations observed in the local heat transfer coefficients. These are shown in Fig. 6. The fluctuations observed in the laminar boundary layer regions of both spheres were equivalent to those of a single sphere. The intensity of the fluctuations was constant through this region and equal on both spheres.

As spheres were brought together, the fluctuations in the heat transfer from the outward surface of the left-hand sphere increased greatly in the turbulent boundary layer region. This increase in activity may be attributed to the lateral flow developed by the attached jet which swept over this surface toward the other sphere. The intensity of the fluctuations was observed to decrease at the closest spacing. This was due to the decrease in the lateral flow with the decrease in the strength of the jet.

The fluctuations on the inward side of the left-hand sphere did not increase as the spheres were brought together, but remained comparable to those in the wake of a single sphere. This is in agreement with the separation of the laminar boundary layer directly into the wake as observed from the local heat transfer measurements.

The fluctuations in the turbulent boundary layer region of the right-hand sphere were smallest in the space between the spheres at wide spacings. This was probably the result of the deceleration of the flow between the spheres. When the jet was attached to the sphere, the fluctuations were greatest in the attached flow region which occurred at spacings closer than 1.19 diameters.

The behaviour of the outward surface of the right-hand sphere was similar to that of the inward surface of the other sphere when the jet flow was strong. The turbulent boundary layer was not as active as that on a single sphere, although the reattachment of the boundary layer was observed to occur after the laminar separation point. The movement of the turbulent separation point was also reflected in the fluctuation data.

At very close spacings, the intensity of the fluctuations on the part of the right-hand sphere closest to the other sphere were lower than those observed at a traverse angle of  $45^\circ$ . This was in part caused by the delay in the separation of the laminar boundary layer of the attached jet, but also by the deceleration of the flow. The fluctuations were largest at a traverse angle of  $45^\circ$ , suggesting a split in the jet as it moved outward from the space between the spheres.

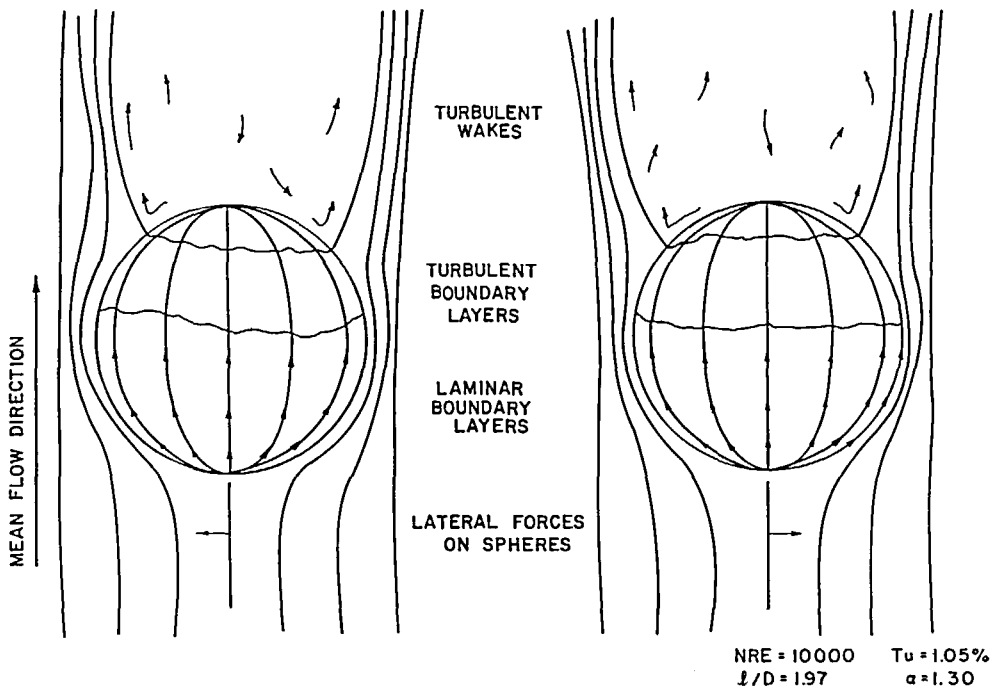


FIG. 7. Schematic of perpendicular flow past two spheres.

These observations may be summarized by compiling detailed descriptions of the flow at several spacings. Figure 7 shows the flow behaviour when the spheres are separated by 1.97 diameters. Although the flow was basically symmetrical, the flow in the space between the spheres was decelerated by the combined blockage of the two wakes. The increased pressure in this region resulted in a net repulsion force which acted equally on each of the spheres. The heat transfer from the turbulent boundary layer of the left-hand sphere indicated the presence of a slight crossflow toward the other sphere. This behaviour became more significant at closer spacings.

Figure 8 shows the flow patterns where the jet attachment was strongest. The flow around the two spheres was deflected to the right by the attachment of a jet-like flow which formed between the spheres. The mechanism of this attachment was the same as that of the Coanda effect. The laminar and turbulent separation points were moved to the rear in the region of the attached jet. The turbulent separation points on the right-hand surfaces of both spheres were shifted forwards by the deflection of the wakes. This deflection was strong enough in the space between the spheres to prevent the reattachment of a turbulent boundary layer after the laminar layer separated from the left-hand sphere. The deflection of the flow gave rise to a lateral force pushing the two spheres to the left. This force was greater than the repulsion, hence the right-hand sphere was pushed to the left. The left-hand sphere experienced both the repulsion and the deflection forces pushing it to the left.

The heat transfer in the region between the two spheres was increased by the deceleration of the mean flow by the wake blockage. The secondary flow induced

by the air moving out, around the spheres, resulted in a thinner boundary layer than would exist in an axisymmetrical flow. Some secondary flow, apparently from the blockage resulting from the deflected wake, was observed on the outward surface of the right-hand sphere. This was not observed on the left-hand sphere whose outer surface was not blocked.

At the closest spacings, as shown in Fig. 9, the deceleration of the flow between the two spheres greatly decreased the amount of flow through the jet. This, in turn, reduced the wake deflection resulting in a gradual return to a symmetrical flow. The secondary flow resulting from the wake blockage became more significant at the closer spacings, giving an increase in the repulsion between the spheres and an increase in the heat transfer from the inward surfaces of both spheres. At the closest spacing, the secondary flow appeared to become sufficiently strong to push the jet out of the space between the spheres giving a 3-dim. jet structure.

The flow around two spheres whose common axis is perpendicular to the mean flow may be concluded to be dominated by the interaction between the two wakes and the free-stream. This interaction occurs at Reynolds numbers of the order of 10 000.

When the spheres are sufficiently close to interact, the combined blockage of the two wakes results in a reduced drag on both spheres, a repulsion between them and higher heat transfer rates. At closer spacings, the jet-like flow bounded by the separated boundary layers become attached to one of the spheres, deflecting the wakes. This deflection results in a lateral antisymmetric force which causes the pair to move to the side. The shift in the wakes causes an increase in the heat transfer as the turbulent boundary layer regions become smaller. At still closer spacings, the blockage

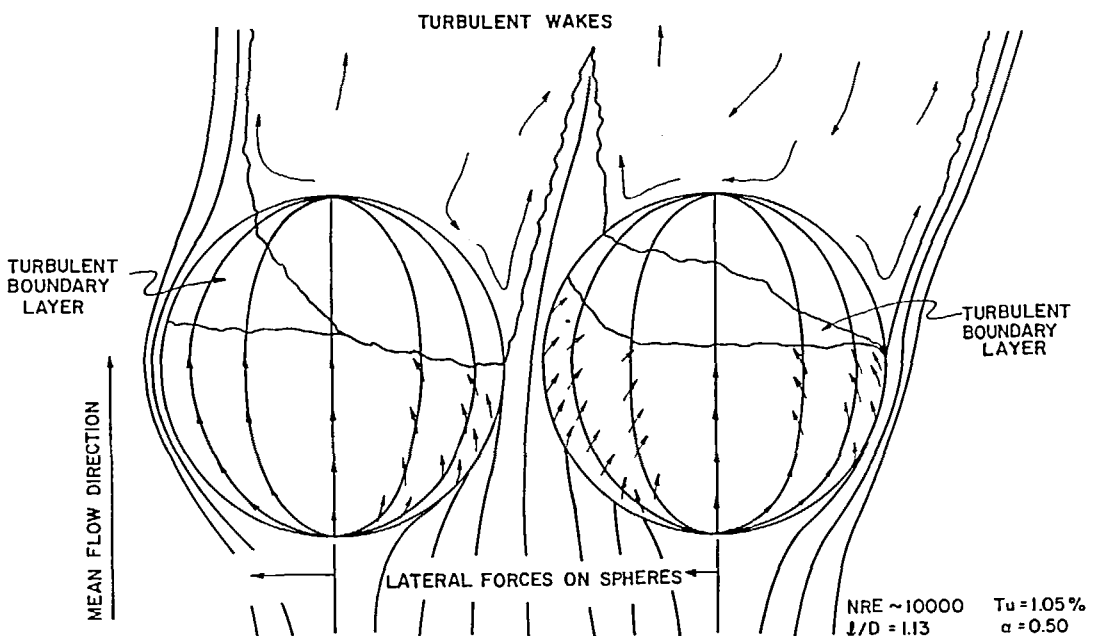


FIG. 8. Schematic of perpendicular flow past two spheres.



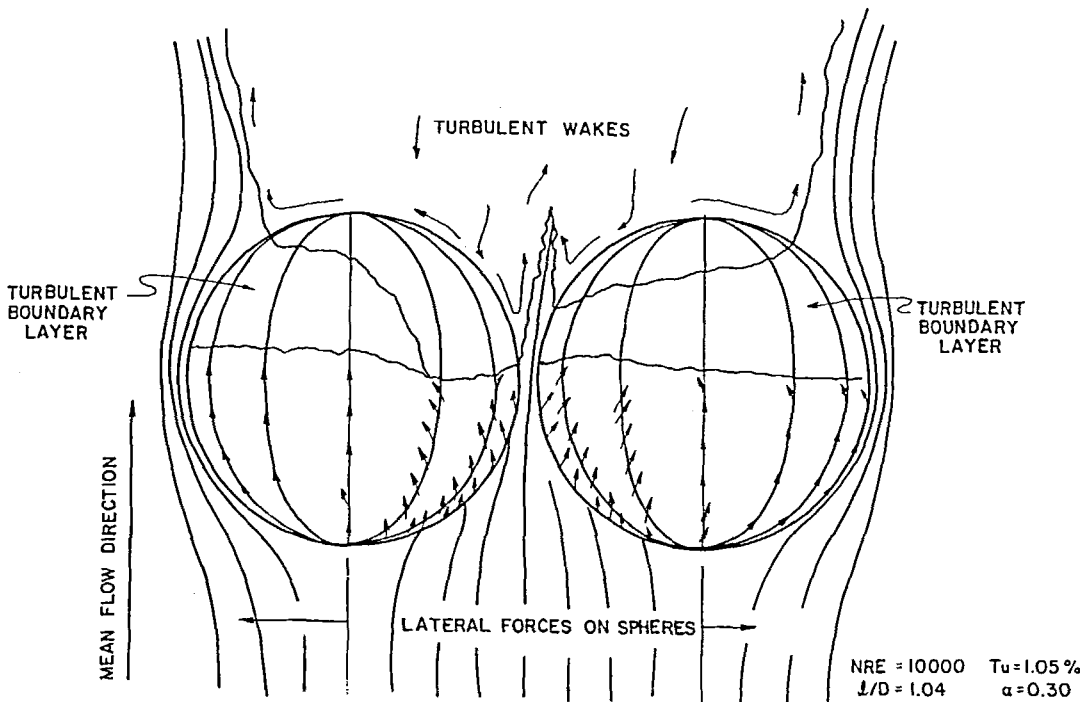


Fig. 9. Schematic of perpendicular flow past two spheres.

due to the wakes and the wall friction between the spheres and the fluid decreases the jet flow sufficiently to result in a partial return to symmetrical flow. The heat transfer from the sphere with the attached jet decreases as the turbulent boundary layer returns while the heat transfer from the other sphere increases due to the increased secondary flow resulting from the increased blockage of the flow between the spheres. To summarize, the forces generated by the interaction are considerable, amounting to 20% of the normal drag force. The increase in the heat transfer is also of comparable magnitude.

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## FLUX THERMIQUES LOCAUX POUR DEUX SPHERES ADJACENTES DANS UN ÉCOULEMENT TURBULENT

**Résumé**—On étudie les interactions pour une paire de sphères en configuration d'écoulement frontal dans une soufflerie verticale. Le nombre de Reynolds est approximativement  $10^4$  avec une intensité de turbulence de 1,1%. L'écartement des centres des sphères varie de 1,04 à 2 diamètres. On mesure à la fois les pressions pariétales et les coefficients de transfert thermique locaux. La technique de mesure donne la composante fluctuante du transfert thermique et la valeur moyenne temporelle. Les résultats indiquent la formation d'un écoulement semblable à un jet entre les deux sphères. A un écartement faible, ce jet reste attaché à l'une des sphères et conduit à un accroissement des forces d'écoulement transversal. Pour des écartements très réduits, le cisaillement accru dans la région entre les sphères s'accompagne d'une diminution de la force du jet et de l'amplitude des forces d'écoulement transversal.

## ÖRTLICHER WÄRMEÜBERGANG AN ZWEI NEBENEINANDER ANGEORDNETEN KUGELN IN EINER TURBULENTEN STRÖMUNG

**Zusammenfassung**—Die gegenseitige Beeinflussung eines quer angeströmten Kugelpaares in einem senkrechten Windkanal wurde untersucht. Die Reynolds-Zahl der Strömung betrug etwa  $10^4$  bei einem Turbulenzgrad von 1,1%. Der Abstand der Kugelmitten wurde zwischen 1,04 und zwei Durchmessern verändert. Es wurden sowohl die örtlichen Oberflächendrücke als auch die Wärmeübergangskoeffizienten gemessen. Das Meßverfahren erlaubt sowohl die zeitlich veränderliche Komponente des Wärmeübergangs als auch seinen zeitlichen Mittelwert zu messen. Die vorliegenden Ergebnisse deuten darauf hin, daß sich eine strahlartige Strömung zwischen den Kugeln bildet. Wenn der Abstand klein ist, liegt dieser Strahl an einer der Kugeln an, was zu Querströmungen und entsprechenden Kräften führt. Bei sehr kleinen Abständen führte die erhöhte Wandschubspannung im Gebiet zwischen den Kugeln zu einer Abnahme der Strahlstärke und der Kräfte, die mit der Querströmung zusammenhängen.

## ЛОКАЛЬНЫЕ СКОРОСТИ ПЕРЕНОСА ТЕПЛА ОТ ДВУХ БЛИЗКО РАСПОЛОЖЕННЫХ СФЕР ПРИ ОБТЕКАНИИ ТУРБУЛЕНТНЫМ ПОТОКОМ

**Аннотация**—В вертикальной аэродинамической трубе исследовались взаимодействия между парой сфер, обтекаемых поперечным потоком. Число Рейнольдса потока составляло примерно  $10^4$  при интенсивности турбулентности в 1,1%. Расстояние между центрами сфер изменялось от 1,02 до 2 диаметров. Проведены измерения локальных значений давления на поверхности и коэффициентов теплопереноса. Используемая методика позволяла измерять как пульсационную составляющую теплопереноса, так и осредненные по времени значения. Результаты показывают, что между сферами образуется струйное течение. При близком расположении сфер струя примыкает к одной из них и создает объемные поперечные силы. При очень близком расположении сфер между ними из-за увеличения напряжения сдвига происходит снижение скорости течения струи и указанных сил.